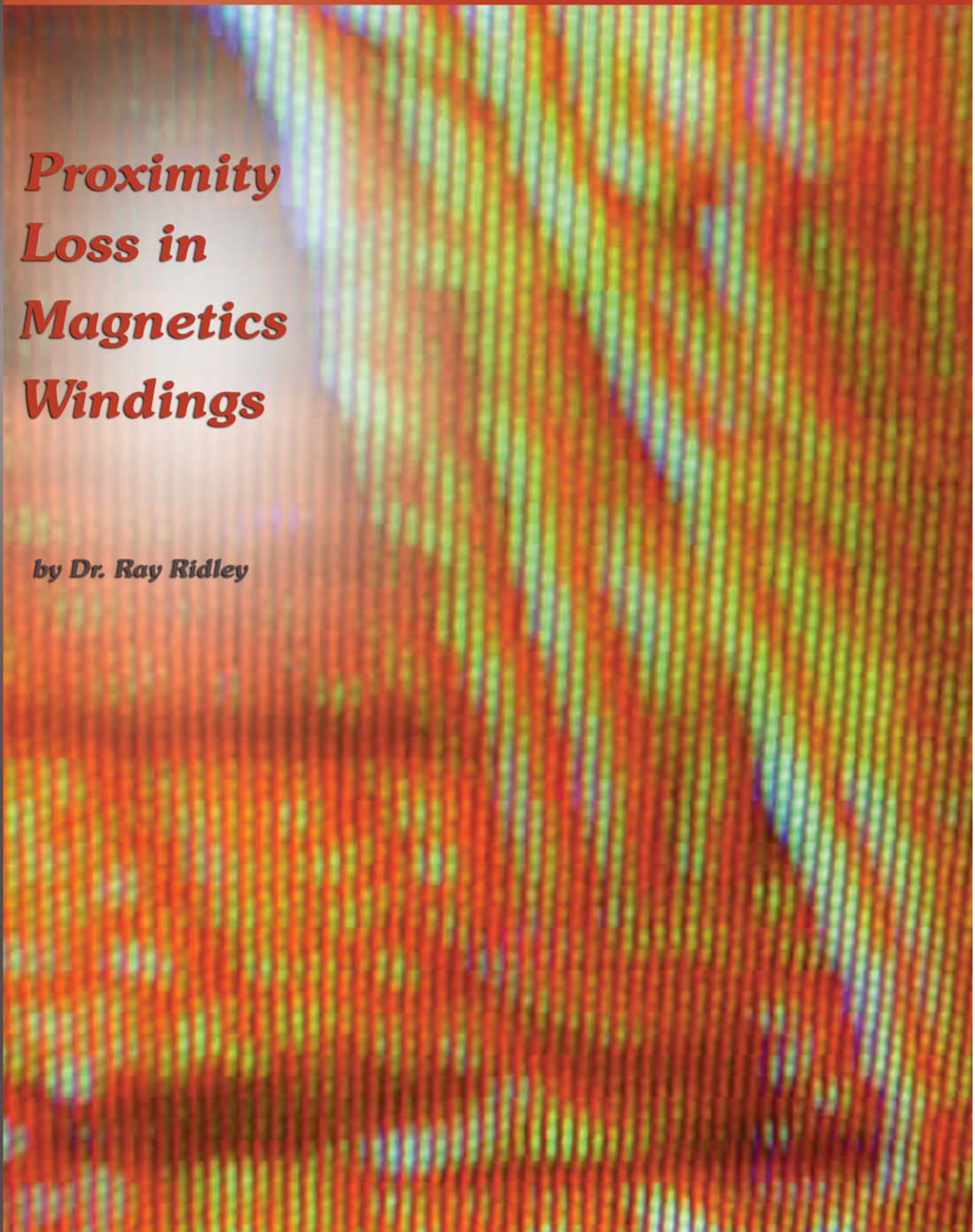
Designers Series XIII

Proximity Loss in Magnetics Windings

by Dr. Ray Ridley



Do you use this equation in your design?

$$P_d = b_w \sum_{i=1}^{n} l_i \frac{1}{h_i \sigma} H_i^2 \left[\left(1+\alpha_i^2\right) G_1(\Delta_i) - 4\alpha_i G_2(\Delta_i) \right]$$

If not, your magnetics are hotter than you think.

We have had many requests over the last few years to cover magnetics design in our magazine. It is a topic that we focus on for two full days in our design workshops, and it has always been my intent to start writing a comprehensive design series for high frequency magnetics.

The problem I've always faced is where to start? There are hundreds of design rules, equations, and tips that make good magnetics work properly. If we started at the beginning, with the fundamental design rules, it would take many issues to get to some of the "good stuff".

So I've decided to work backwards– starting with the most complex topic in magnetics design that every engineer faces with his or her system. This will serve two purposes– to help you understand the magnitude of the problems faced in seemingly simple designs, and to point out to those who think that magnetics is easy how involved and analytical it must be to achieve good working designs.

Proximity Loss: What is it?

Any time a conductor is inside a varying magnetic field, currents will be induced in the conductor and these induced circulating currents are referred to as eddy currents. This is an unavoidable fact of electromagnetic theory. The eddy currents make no net contribution to the overall current flow through a wire, but merely serve to redistribute current in a conductor, and increase power dissipation.

Most designers are aware of skin effect, and use rudimentary calculations to estimate if their designs have skin effect issues. Skin effect is the tendency of the current in a wire to flow towards the surface of the wire more and more as the frequency of excitation is increased. Figure 1 shows an isolated wire in free space carrying an ac current.

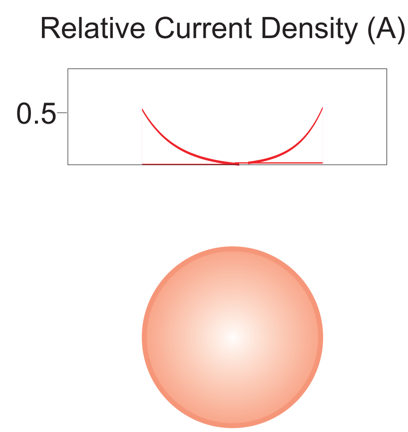


Figure 1: Current distribution in an isolated conductor in free space.

The current density is greatest at the surface, and it decays exponentially inside the wire, reaching a value of 1/e times the surface current density at the skin depth. This happens because nature tries to minimize the energy storage due to the high-frequency current. A large diameter wire has smaller self-inductance than a small-diameter wire, and the flow of current in the surface due to skin effect minimizes the inductance. The skin depth is easily calculated from:

$$\delta = \sqrt{\frac{2}{\omega\mu_o\sigma}}$$

with $\mu_0 = 4\pi \times 10^{-7}$, ω is the operating radian frequency, and σ is the conductivity of the winding material. (copper conductivity = 5.8×10^7 at room temperature.) All units are SI.

This equation is easy to apply. At 60 Hz, the skin depth is 8.5 mm. If you don't use a conductor thicker than 8.5 mm, the skin effect on dissipation will be small. At 100 kHz, the skin depth is 0.2 mm, or about half the diameter of a #27 wire. If you don't use a bigger gauge than this, the skin depth calculation says you are OK.

But are you? Unfortunately not. Unless you build your magnetic with a single strand of wire in free space, which is the situation in which the skin depth equation applies directly, your actual copper losses will usually be much higher than predicted.

This is due to proximity loss–the effect of putting a conductor in a field generated by other conductors in close proximity to it. Proximity loss is always more than skin effect loss, and will affect the design and temperature rise of almost all high-frequency magnetics. Even if you wind an inductor with just a single turn, the fact that the turn is shaped around the bobbin affects the current distribution in the conductor, as shown in Figure 2.

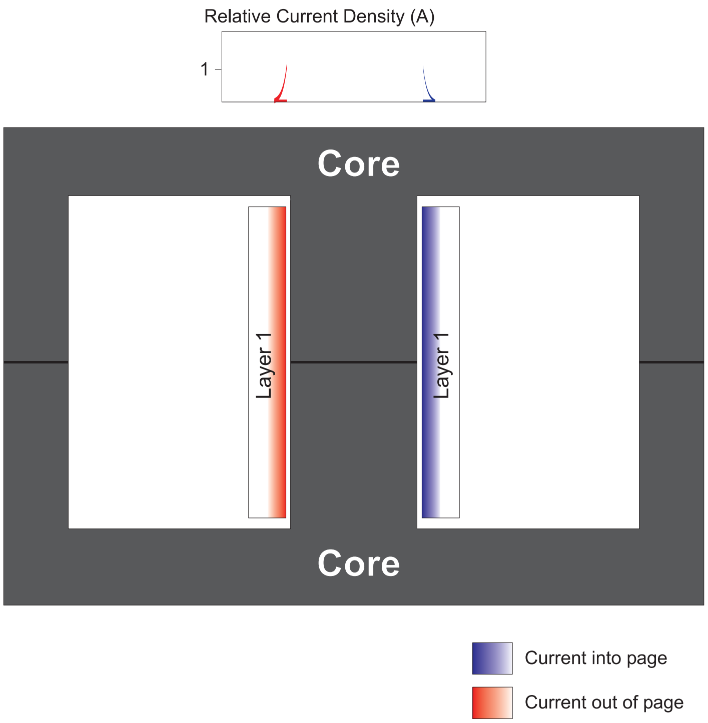


Figure 2: Current distribution with single-layer inductor winding

This picture shows the cross section of an inductor with the center leg of the core, and a single layer turn. The blue shading represents current flowing into the plane of the page, and the red shading the current flowing out of the page at any given instant. The intensity of the shading represents the current density at that part of the conductor.

Notice that the current no longer flows along both surfaces, but moves towards the inside surface only. This again due to the tendency of the high-frequency current to arrange itself to minimize the inductance created. The smaller the area enclosed by the current, the smaller the inductance.

The tendency of the current to move towards the surface causes an increase in ac resistance, but we can no longer use a simple equation to calculate this effect. We need to move on to more complex analysis to solve the problem.

Proximity Loss Calculation

All magnetics structures are complex three-dimensional objects that defy any exact analytical modeling. You can, if you like, apply Finite Element Modeling software to try to analyze the complete three-dimensional fields and currents, but this is a very time-consuming activity. Plus, it will only give you results for a finished design, not insight into how to create a design.

As with any aspect of engineering, we don't just need numerical solutions, but *symbolic* analytical results are needed to yield design rules and procedures. For magnetics, we have to simplify the three-dimensional structure of winding design, all the way down to a model with only one dimension of freedom. In order to obtain analytical results, we must assume that the winding layers of an inductor or transformer are continuous sheets of conductor, with no variation in the x-y direction, and variation as we go through layers of conductors in the z-direction. This is, of course, a gross oversimplification, but the results yielded with this approach provide us with powerful design guidelines to build better magnetics.

Figure 3 shows the cross section of an inductor with five layers of rectangular winding around the center leg of an E-E core. Electric fields will be relatively uniform for the part of the winding away from the edges, and we extend the uniform field assumption all the way to the edge in order to find an analytical solution to the currents in the winding layers.

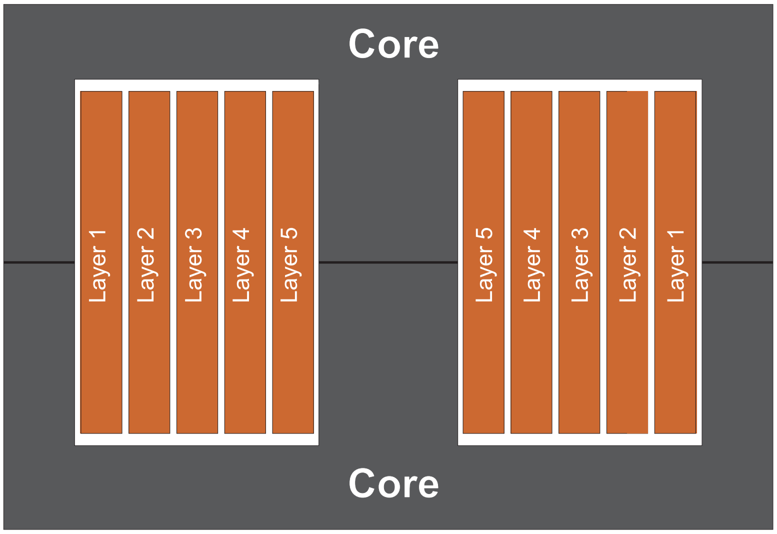


Figure 3: Cross-section of an inductor with a five-layer winding

Figure 4 shows a cross-section of one side of the bobbin of a five-layer structure. The windings are truncated in the horizontal plane, but are assumed to have no end effects, leading to uniform fields, H_1 through H_6 between the layers of windings. The windings are assumed to be a continuous uniform cross-section of copper, as would be obtained with a flat foil. (In many designs, this layer may be formed with individual wires, and these must be approximated to a foil sheet, a step outside the scope of this article.)

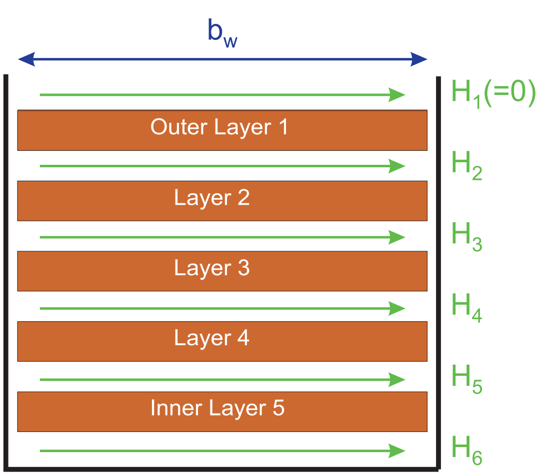


Figure 4: Multilayer winding structure with variation in the z-direction only.

The simplification to a one-dimensional variation only is needed in order to arrive at an analytical solution to Maxwell's equations. It is not possible to arrive at closed-form solutions of a situation more complex than this.

Dowell's Equation

Even this simple situation is a formidable problem to solve. Fortunately for modern engineers, it has already been done, so all we have to worry about is understanding and applying the results.

At the heading of this article is a form of Dowell's equation. This gives the power dissipation P_d, of a winding by summing up the power loss through each of the layers from 1 to n. It is a crucial analytical result that lets us calculate the dissipation of a magnetics winding at frequencies other than dc. It is a very intimidating equation, even more so when you examine the terms in it such as:

$$G_1(\Delta) = \Delta \frac{\sinh 2\Delta + \sin 2\Delta}{\cosh 2\Delta - \cos 2\Delta}$$

$$G_2(\Delta) = \Delta \frac{\sinh \Delta \cos \Delta + \cosh \Delta \sin \Delta}{\cosh 2\Delta - \cos 2\Delta}$$

These terms are combinations of sinusoidal functions, which we use all the time, and hyperbolic sine and hyperbolic cosine functions which, most likely, you haven't seen since college calculus.

The term Δ is the ratio of the height of a winding layer, divided by the skin depth for the given frequency of analysis.

$$\Delta_i = \frac{h_i}{\delta}$$

where h_i is the height of layer i and the skin depth, δ, is given earlier in this article. In other words, if the winding layer height is equal to 2 skin depths, $\Delta = 2$.

At this point, you may be feeling overwhelmed by the mathematical expressions. You should be– these are not easy concepts to think about, or easy equations to grasp. Like many power supply engineers, I spent a large part of my career aware of the problems of proximity loss, but unable to apply the results of analysis due to a lack of time to spend to try and understand it all. The texts and papers containing this material are very advanced, and you need to set aside a substantial amount of time with no interruption to make good progress.

The dedicated engineers amongst you will want to press on and try to apply this equation. For this, you will need the following additional information:

The field at the boundary of each layer i is given by:

$$H_i = \frac{N_i I_i}{b_w}$$

where N is the number of turns forming the layer (which may be formed of N multiple adjacent conductors), and I is the current through each turn. The width of each layer of conductor is denoted by b_w, and the length of a turn on layer i is given by l_i.

The final term you need to know about is α_i, which is the ratio of the fields on either side of layer i. The convention for calculating this is to order the fields such that the smaller one is in the numerator. For the 5-layer inductor example, the calculations would be:

$$\alpha_1 = \frac{H_1}{H_2} = 0 \quad \alpha_2 = \frac{H_2}{H_3} = \frac{1}{2} \quad \alpha_3 = \frac{H_3}{H_4} = \frac{2}{3} \quad .... \quad \alpha_4 = 0.75 \quad \alpha_5 = 0.8$$

(Note: one indication of how "effective" a winding is can be assessed from the value of this term. As it approaches unity, that means that the fields on either side of the winding are minimally affected by the current through that winding. That means you have a piece of conductor sitting in a field with induced eddy currents in it, and the winding will be inefficient.

The most effective windings have an associated α value of -1. This can only be achieved in transformer windings, and occurs when the current through a layer reverses the field from one side to the other. This is similar to an isolated conductor in free space which experiences only skin effect.)

With this information, you can now work on solving the equations. The simplest way to set up the solution to this equation is in either MathCad or Excel. The complicated terms G_1 and G_2 are merely constants for a given layer height and skin depth. (The results can be simplified if the result is normalized for a 1 A current in terms of the dc resistance of the wire.)

If you don't have the few days necessary to experiment with this, the results have all been programmed into ***POWER 4-5-6***, and you can use this as a handy calculator to find the ac resistance of a winding very quickly.

Proximity Loss Results for 5-Layer Winding

Once you have programmed equation 1 into your favorite software, the input variables for calculating the ac resistance in terms of the dc resistance are the frequency of operation, the height of the winding layers, and the number of layers of winding used. From this, you can calculate the resistance ratio for each layer of the winding, as shown in Table 1.

The results are very surprising if you are not familiar with the severe effects of proximity loss. The first row shows the results for a layer with a foil 0.3 mm in height (about the same as the diameter of a 30 AWG wire), and with a frequency of 100 kHz. The ratio of the layer height to skin depth is 1.46, certainly a reasonable choice according to skin depth design rules.

Notice how rapidly the ac resistance increases from one layer to the next. The innermost winding layer has 27 times the resistance that would be calculated or measured at dc. The average increase over the 5 layers is 11.6 times.

It is common when an inductor is running too hot to increase the thickness of the copper used. If it is doubled to 0.6 mm (24 AWG wire diameter), the average increase in resistance is 51 times. The inner winding (which is also the most difficult to extract heat from) has 123 times increased resistance. Increasing further to 1.1 mm almost doubles these numbers again.

The rearrangement of current in the windings for this case is more complex than for a single layer. Figure 5 shows the current distribution in the winding layers. On the outermost layer, the current moves toward the inside of the conductor. The net current through this turn is normalized to 1 A.

Layer Number (1 = outer layer)

Layer Height	1	2	3	4	5	Overall R_{ac}/R_{dc}
0.3 mm Δ = 1.46	1.35	3.91	9.04	16.74	27.01	11.6
0.6 mm Δ = 2.80	2.81	14.87	39	75.19	123.45	51.1
0.9 mm Δ = 4.33	4.33	22.25	58.1	111.86	183.55	76.0
1.1 mm Δ = 5.38	5.38	26.95	70.09	134.8	221.08	91.7
0.3 mm 2 layers	1.35	3.91	-	-	-	2.6

Table 1: AC to DC resistance ratios at 100 kHz for 5-layer windings with different thickness

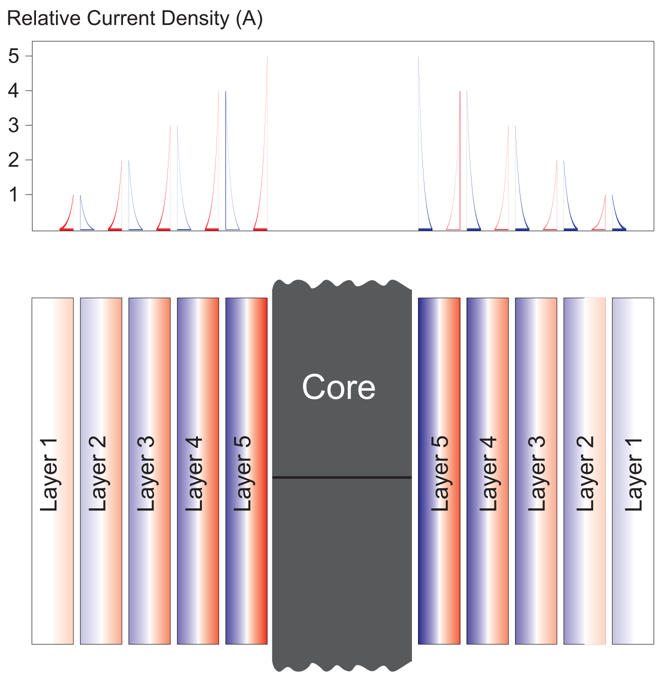


Figure 5: Current distribution with five-layer inductor winding

The second layer has two current components– one on the outside of the conductor which mirrors the adjacent current density in the outermost turn, and flows in the opposite direction to the net current through the turn. This is Mother Nature arranging the currents to cancel high-frequency magnetic fields. The second component of the current is on the inside of the layer, which has twice the amplitude of the outer current. The net current through the turn must, of course, be the same as for the outermost layer since they are series turns of the same inductor.

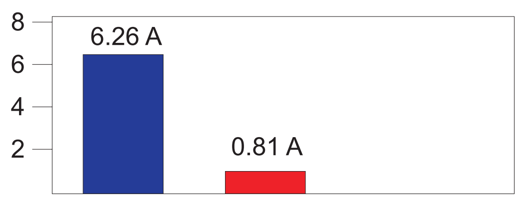
The third layer again has two components– an outer reverse current with a normalized amplitude of 2 A to mirror the turn outside it, and an inner current with a normalized value of 3 A. This continues through the remaining layers.

The most powerful way to reduce the proximity loss resistance in an inductor winding is to reduce the number of layers. The final row of the table shows that the overall ac resistance is only 2.6 times the dc resistance with just two layers, versus 27 times with five layers. In this case, it is much better to use a finer gauge wire that can fit the needed number of turns in two layers, versus a bigger wire that requires five layers.

Choosing the Right Conductor Size

Before assessing which thickness copper is best for a given application, you must also assess the current waveforms which will be applied though the winding. For an output inductor on a dc-dc converter, there will be a strong dc component of current, as shown in Figure 6.

Despite the relatively large amount of ripple shown in the waveform, the dc portion of the current accounts for 6.26 A, but the ac portion is only 0.81 A rms. For most proximity loss analysis, it is common to split the current into just two parts– the dc and the ac. The combined ac component is the rms value of all of the Fourier components other than dc. The dc current is then used to calculate the dc resistive loss. The ac current is assumed to all be at the switching frequency, and used with the calculated ac resistance at that frequency to estimate the ac conduction loss. In many inductor design cases, the ac resistance can be very high, but the overall contribution to loss can be quite small due to the large dc component. This allows us to design multi-layer inductors effectively despite the large proximity effect resistances.



I_{dc} $\quad I_{ac} = \sqrt{I_1^2 + I_2^2 + I_3^2 + I_4^2 + ...}$

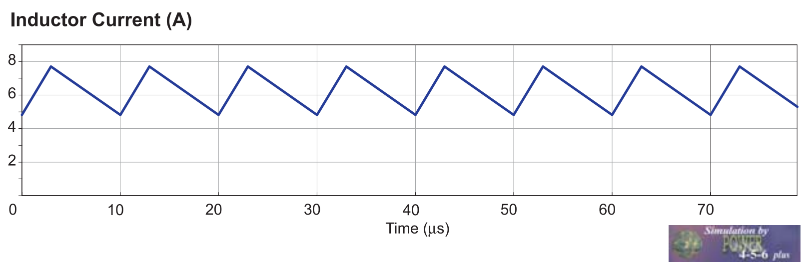


Figure 6: Inductor current waveform and components

For more precise proximity loss estimation, the current waveform should be split properly into its full spectrum of frequency components, and each of these used with the corresponding ac resistance calculation to find the more accurate result. For converters with reasonable duty cycles close to 50%, combining the ac currents into a single expression is usually sufficient. If your converter has very narrow current pulses, the more involved calculation will yield significantly higher and more accurate results.

Transformer Winding Proximity Loss

Transformer windings typically have much stronger ac components in the current waveforms, as shown in Figure 7. For this reason, we avoid the use of multiple layers of primary and secondary windings in transformers.

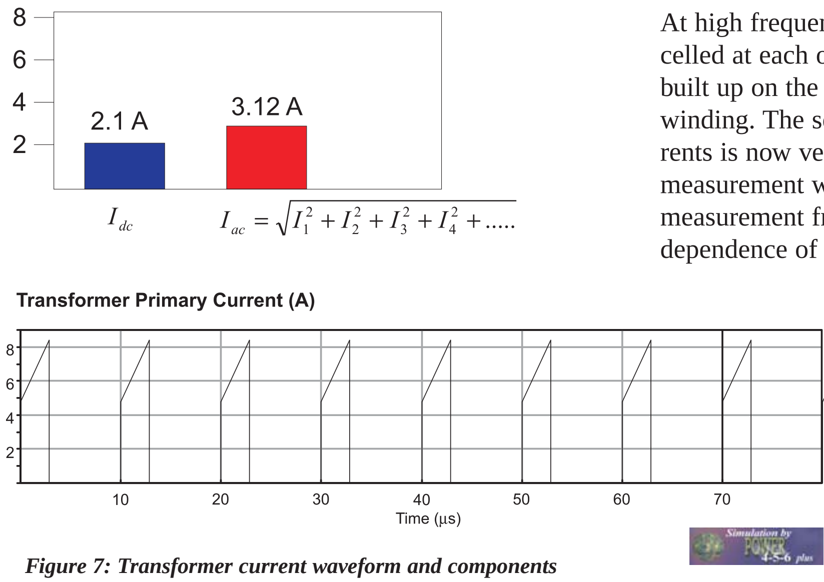


Figure 7: Transformer current waveform and components

Fortunately, we have more design flexibility when making high-frequency transformers than we do when making inductors. In an inductor, all of the current flows in the same direction, and our only design freedoms are the number of layers and conductor thickness.

In a transformer, primary and secondary windings conduct in opposite directions. If we arrange a single layer of transformer primary next to a single layer of transformer secondary, the currents flow as shown in figure 8. At high frequencies, the currents flow close to the surface, in an effort to cancel the magnetic field from adjacent windings. The calculations for proximity loss of this single layer design are the same as for an inductor.

When we add more layers to a primary or secondary, we should always try to avoid making multiple layers, as for the inductor, since this will lead to higher conduction losses. Figure 9 shows how the currents are distributed in a five layer primary winding. This is identical to the inductor current windings, with the exception of the current in the secondary, which is in the opposite direction. The secondary winding acts as a single layer winding. It does not make any difference to this winding whether the primary is constructed of single or multiple layers.

An interesting side note arises out of this particular arrangement of currents in multi-layer transformer windings. At low frequencies, the currents are distributed evenly through the layers. Leakage inductance, which is determined by the separation of primary and secondary currents, is large when measured at low frequencies.

At high frequencies, all of the primary fields are cancelled at each of the surfaces until a current of 5 A is built up on the inner surface, close to the secondary winding. The separation of primary and secondary currents is now very small, and the leakage inductance measurement will drop substantially as we increase the measurement frequency up to this point. This frequency dependence of leakage inductance is very important to consider when designing and characterizing high-frequency magnetics.

Also, if you see a substantial change in leakage inductance measurements as you approach the switching frequency of your converter, it is a good indication that you will have significant proximity loss in your transformer.

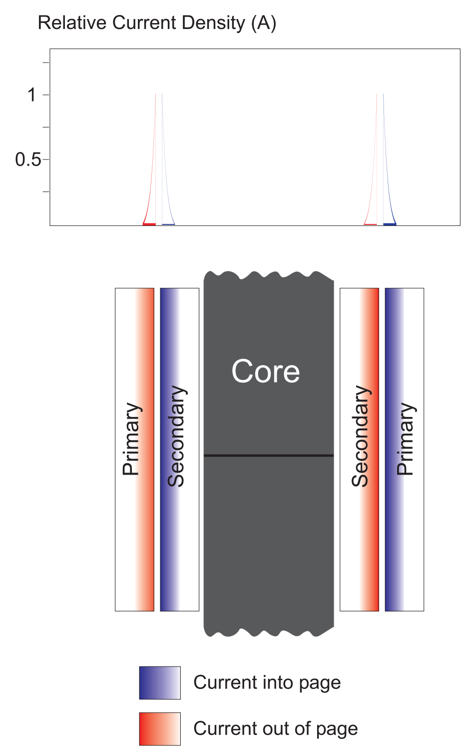


Figure 8: Current distribution with single-layer primary and secondary transformer windings

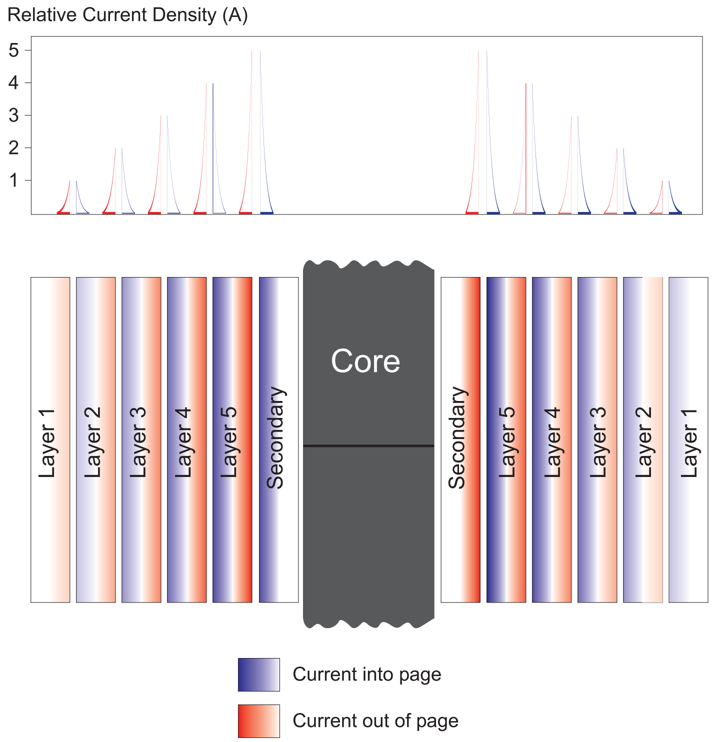


Figure 9: Current distribution with five-layer primary and single secondary transformer windings

Conclusions

Out of all the detailed papers on magnetics design and proximity loss, one simple rule always emerges: keep conductors out of strong high-frequency magnetic fields. There are many ways to achieve this. The simplest inductors are constructed of single-layer designs where the ac losses are minimized. This is essential for inductors that have strong ac components of current relative to dc currents. If you must have more than one layer, due to the number of turns required, they should be the right thickness to provide the optimum balance of dc and ac resistance for your particular waveforms. There are no closed-form formulas for optimizing this due to the wide variety of variables involved. We recommend returning to the basic proximity loss equation to find the right balance for your design.

Transformers are designed the same way– keep to no more than a single layer design, and if you need more layers to accommodate turns, try to

Interleaved Transformer Windings

In some low-cost designs, we are forced to keep primary and secondary windings separate due to safety and spacing requirements that do not allow multiple insulation systems. However, an ideal transformer deliberately introduces multiple primary and secondary windings, interleaved with each other, to reduce field strengths and minimize proximity loss effects.

Figure 10 shows how splitting the primary into two parts, one on either side of the secondary, can lower the loss in a transformer.

Each of the primary layers now carries just half the net current, and has proximity loss according to the single-layer solution. This is a dramatic improvement over building the two primaries in succession, which could end up with as much as five times the loss.

Even the secondary winding, which remains unchanged in terms of its construction in the transformer, experiences a drop in proximity loss. The current in the secondary now flows equally in both surfaces, and much better utilization of the conductor is achieved.

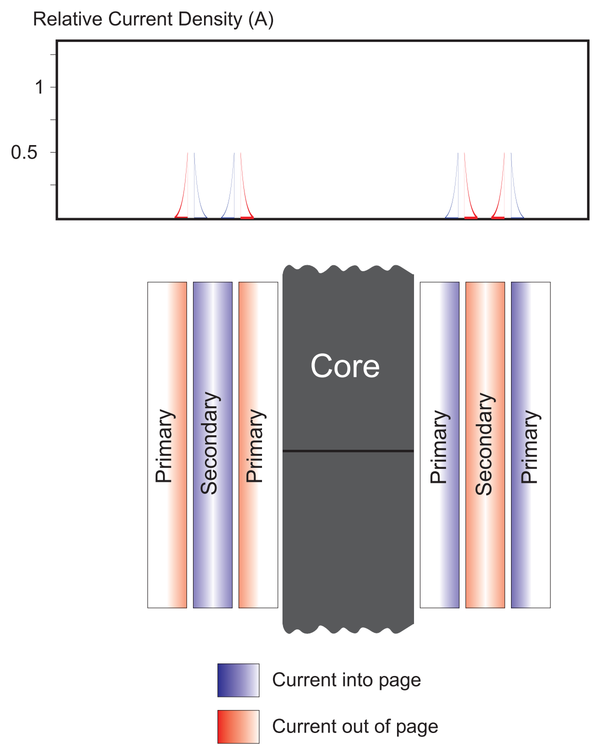


Figure 10: Current distribution with single-layer secondary and split primary transformer winding

build them interleaved with secondaries to minimize the accumulated magnetic fields. This is not always practical with conventional isolation techniques due to the expense of multiple safety boundaries.

Planar transformers lend themselves naturally to multiple interleaving. Each layer of a pc board has limited turns count and current-carrying capability, and it is commonplace to have numerous primary and secondary layers to build power transformers. While planars can work well, they are not always the most cost-effective solution. Many power supply manufacturers cannot even afford to go beyond a single layer PC board, and conventional wound magnetics are still the lowest cost solution. Prices on standalone planars have dropped significantly recently, however, and should be considered for custom designs. Manufacturers such as Payton magnetics can provide custom quick-turn designs for your applications.

There are many more aspects to proximity loss that are not covered in this article, such as end effects, core gap effects, and different wire shapes, that complicate design further. We will cover some of these effects in upcoming issues of Switching Power Magazine.